This article was downloaded by: [75.94.224.40]
On: 07 July 2013, At: 17:26
Publisher: Taylor & Francis
Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK

Impact Assessment
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tiap19

MARKOV FORMULATION OF CROSS-IMPACT ANALYSIS FOR IMPACT ASSESSMENT AND FORECASTING

James J. Swain [a], Troy Halverson [b], Frederick A. Rossini [c], Alan L. Porter [d] & Huaidong Xu [d]

[a] Lionheart Publishing, 2555 Cumberland Parkway, Ste 299, Atlanta, GA, 30339

[b] Coca-Cola USA, One Coca-Cola Plaza, Atlanta, GA, 30301

[c] George Mason University, Fairfax, VA, 22030-4444

[d] School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, GA, 30332-0205, USA

Published online: 06 Feb 2012.

To cite this article: James J. Swain , Troy Halverson , Frederick A. Rossini , Alan L. Porter & Huaidong Xu (1991) MARKOV FORMULATION OF CROSS-IMPACT ANALYSIS FOR IMPACT ASSESSMENT AND FORECASTING, Impact Assessment, 9:3, 55-74, DOI: 10.1080/07349165.1991.9725718

To link to this article: http://dx.doi.org/10.1080/07349165.1991.9725718

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the "Content") contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms & Conditions of access and use can be found at http://www.tandfonline.com/page/terms-and-conditions

Downloaded by [75.94.224.40] at 17:26 07 July 2013

MARKOV FORMULATION OF CROSS-IMPACT ANALYSIS FOR IMPACT ASSESSMENT AND FORECASTING

James J. Swain, Troy Halverson,
Frederick A. Rossini, Alan L. Porter, Huaidong Xu *

Downloaded by [75.94.224.40] at 17:26 07 July 2013

INTRODUCTION

Cross-impact analysis (C-I) is a tool that has been used in systems analysis, technology forecasting (Martino, 1983) and technology assessment (Porter et al., 1980). C-I attempts to determine the impact of the occurrence of sets of trends and/or events on other trends and/or events.

Variations of C-I are widespread; Halverson et al., 1989 and Xu, 1990, review these. In its simplest form it consists of a matrix, one dimension of which is a set of factors of interest, and the other a set of factors that may influence them. Entries in the cells of the matrix are estimated by qualitative judgment to indicate the importance of the interactions between the elements of the columns and the rows.

For quantative applications the cells usually contain conditional probabilities that represent interactions between row and column entries. It is generally assumed that conditional probabilities (i.e., the probability of E_1 given that E_2 occurs) can be estimated more accurately than marginal probabilities (i.e., the probability of E_1). Therefore, several version of C-I use the estimates of the conditional probabilities to adjust the marginal probabilities. For instance, the marginal probabilities can be adjusted using a Monte Carlo simulation (c.f., Enzer and Leschinsky, 1986; Porter,

*Swain is with Lionheart Publishing, 2555 Cumberland Parkway, Ste 299, Atlanta, GA 30339; Halverson, Coca-Cola USA, One Coca-Cola Plaza, Atlanta, GA 30301; Rossini, Vice Provost, George Mason University, Fairfax, VA 22030-4444; Porter and Xu, School of Industrial & Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA.

Downloaded by [75.94.224.40] at 17:26 07 July 2013

et al., 1980) or in closed form (Halverson et al., 1989). However, the conditional probabilities are not ordered by their time of occurrence, that is, they are not time-ordered.

Reitman et al. (1985) presented a technology forecast of AI (artificial intelligence) developments and how three Air Force aims (or targets) depended on them. The bottom line of this AIM-TECH forecast was to show the relative need for R&D investment in these technologies in relation to development time-lines and the likelihood of achieving the three aims. We focus on a few aspects of the AIM-TECH forecast to illustrate C-I.

One target was to use AI and related technologies to complement pilot decisionmaking capabilities in difficult flight situations. One key capability of several needed to fulfill the AIM-TECH pilot/aircrew automation scenario entails "Planning Against Adversaries." This capability depends on two other capabilities – a "Common Sense System" and "Analogical Reasoning." AIM-TECH experts estimated:

1. Probabilities of attaining each of these three capabilities;
2. The minimum/maximum time periods required to attain them; and,
3. The time profile of additional resources required to accomplish them.

Investment strategy in light of these aims was then considered.

In the following example, AIM-TECH information is used purely for illustrative purposes. Only a few of the technologies are considered over a 10-year period, and a number of arbitrary assumptions beyond the AIM-TECH data are made. Consider:

- ANA – attaining an AI system that can reason by analogy (event).
- COM – attaining an AI system that embodies "common sense" (event).
- ADV – attaining an AI system that can plan action against an adversary (event).

ANA, COM, and ADV cannot unoccur – once they have been accomplished.

Downloaded by [75.94.224.40] at 17:26 07 July 2013

THE TRADITIONAL CROSS-IMPACT GAME

Input data for the traditional C-I game consist of expert estimates of the probability that certain events occur. We will briefly review these data requirements to establish notation and to show how these probabilities are modified in the markov formulation. The occurrence of event i is denoted by E_i and its non-occurence by $\bar{E}_i$. Three types of information are needed for the game: **Marginal probabilities** – The estimated probability that an event occurs, $p_i \equiv p(E_i)$; **Occurence probabilities** – the estimated conditional probability that an event occurs given that another event occurs, $p_{i:j} \equiv p(E_i|E_j)$; and **Non-occurence probabilities** – the estimated conditional probability that an event occurs given that another event *has not* occurred, $q_{i:j} \equiv p(E_i|\bar{E}_j)$.

The notation $p_{i:j}$ is used as shorthand for the probability of E_i given E_j, while $q_{i:j}$ is used as the probability of E_i given $\bar{E}_j$. For the AIM-TECH 3-event illustration E_1 stands for ANA, E_2 stands for COM, and E_3 represents ADV.

In this formulation, the diagonal occurrence probabilities must equal 100% and the diagonals of the non-occurrence matrix must be 0%. In general, the expert will have to provide $(2n^2 - n)$ probability estimates to play the game. The data requirements for estimating non-diagonal probabilities may be reduced within each conditional matrix by using the following general rule for any two pairs of events:

$$p_{i:j} = p_{j:i} p_i / p_j \,, \text{for} \quad p_j \neq 0 \tag{1}$$

This rule provides a way of deriving consistent probabilities on one side of the diagonal given the conditional probabilities on the other side. Thus, at minimum, the expert must enter $\frac{1}{2}n(n-1)$ for each conditional matrix, or roughly $n^2/2$ probabilities for the whole game. Alternatively, all probabilities could be estimated.

After all are estimated, the C-I game is played using Monte Carlo simulation. The occurrence and non-occurrence matrices only specify joint probabilities of one event conditional on one other event (called "second-order" conditionals). Therefore, to perform the simulation, additional probabilities are required , (e.g., the conditional probability of an event occurring given two or more other events). Instead of asking for third and higher order contitionals, such as $p_{i:jk}$ and $p_{i:jkl}$ which would require many difficult, and potentially arbitrary, estimates – it is usually assumed that higher order conditionals can be approximated by averaging second-order probabilities. That is, given events 1,...,n:

$$p_{i\ :1,\ldots,i-1,i+1,\ldots,n} \approx \frac{1}{n}\sum_{j\neq i}^{n} p_{i:j} \tag{2}$$

Downloaded by [75.94.224.40] at 17:26 07 July 2013

A similar averaging is used for higher-order non-occurrence probabilities, $q_{i:jkl}$, and probabilities such as $p(E_i|E_j\bar{E}_k) \approx \frac{1}{2}(p_{i:j} + q_{i:k})$.

Time Dependence and Traditional C-I Approaches

In the form just illustrated, C-I is expressed in terms of time-independent conditional probabilities. For instance, $p_{i:j}$ says nothing about whether one event precedes the other or if they occur simultaneously. It merely states the probability of observing E_i given that E_j occurs. Thus, the forecast of the C-I game is time-independent. More realistically, the analysis should include time-dependence information. Certain events may have strong immediate effects that subside with time – e.g., a local embargo – while other events take time to be felt – e.g., accumulation of debt. In other cases, there may only be a "window of opportunity" for a technology to take hold so that the likelihood of other events would change over time as well. The temporal information in Table 1 (based on AIM-TECH) is not captured by traditional C-I. For example, the table shows that ANA can only occur through 1994, whereas the earliest occurrence possible for ADV is 1994. Were one only interested in events from 1995-2000, traditional C-I misses the dynamics of this period.

In place of conditionals like $p_{i:j}$, the C-I game can be restated in terms of time-dependent probabilities, like $p(E_i^{t+1}|E_j^t)$, that incorporate both time dependence and ordering by specifying the probability of E_i at time t+1, given that E_j has occurred by time t. This opens up the possibility for a richer C-I analysis.

Analysts have introduced temporal dynamics into various C-I formulations (Alter, 1979, reviews several approaches). Umpleby (1969) considered probabilities as functions of time while Ketchel and Dolan (1976) considered cumulative probability distributions over time. Bloom (1977) offered a time-dependent model; Enzer and Alter (1978) truncated the future into a few time domains and utilized time interval simulation.

Table 1: Illustrative Probalistic Forecast of Events ANA, COM, and ADV. (Entries are %'s)

	1984	'85	'86	'87	'88	'89	'90	'91	'92	'93
ANA	0	0	0	0	10	20	31	43	56	70
COM	0	0	0	0	0	0	0	0	1	6
ADV	0	0	0	0	0	0	0	0	0	1
	'94	'95	'96	'97	'98	'99	'00	'01	'02	
ANA	85	85	85	85	85	85	85	85	85	
COM	15	28	45	65	65	65	65	65	65	
ADV	1	3	6	9	13	18	24	30	37	

Downloaded by [75.94.224.40] at 17:26 07 July 2013

One approach is to truncate the total time period into a small number of intermediate periods. Separate C-I matrices then can be constructed for each period (Amara, 1972; Martino, forthcoming). The effort required is reduced by noting that only certain events are plausible candidates to occur in certain time periods. For instance, suppose that the 1984-2002 period in Table 1 was broken into three periods: 1984-90, 1991-96, and 1997-2002. In the first period, only ANA is plausible so no conditional probabilities are involved. For the second time period, only ANA could already have occurred, so one might orchestrate a C-I in which only conditionals on ANA are considered. Note, as with other CI-formulations, there are tough modeling choices – whether to consider "within period" cross-impacts. In the third period, ANA is no longer a candidate.

REFORMULATION OF C-I AS A MARKOV PROCESS

C-I can be reformulated as a Markov process. Given the mathematical tools available for Markov analysis, one may answer a variety of forecasting questions, including the probabilities (and variances) of events over time, sequences of events over time, and expected times of occurrences. Howard (1971) provides a general introduction to Markov processes. Eymard (1977) and Kaya et al. (1979) have explored Markov approaches to C-I. We will compare approaches as we develop our Markov model.

A Markov process is a system of states governed by a matrix of transition probabilities. These are the probabilities that the system will go from any state $N_t = i$ to some other state $N_{t+1} = j$ during the next time step. Markov analysis assumes that the transition probabilities are only dependent on the current state, in other words, the Markov process has no memory. For n events, there will be 2^n states which define a $2^n x 2^n$

Downloaded by [75.94.224.40] at 17:26 07 July 2013

Markov transition matrix. Each of the numbered states represents a distinct scenario – a distinct combination of occurrences or non-occurrences of the n events.

The scenario, $\{E_1 \ldots E_n\}$, is cumbersome to write, so we adopt the following shorthand notation. Define the vector, $X = (X_1, \ldots, X_n)$, whose components are $X_i = I_{E_i}(A)$, where A is any scenario and the indicator function I(.) is

$$I_{E_i}(A) = \begin{cases} 1 & if \quad A \subset E_i \\ 0 & if \quad A \not\subset E_i \end{cases} \tag{3}$$

X is a vector, one of whose realizations is $x = (1, 0, 0)$, which would be equivalent to $A = \{E_1 \bar{E}_2 \bar{E}_3\}$ and $x = (100)$ for simplicity. The states can be numbered arbitrarily or through a function such as $N(X) = \sum_{i=1}^{n} X_i 2^{n-i}$.

As an example, below is a transition matrix for a system of 3 events. For three events there are $2^3 = 8$ states which define an 8 x 8 transition matrix. The Markov transition matrix probabilities are represented by

$$p_{t:j}^{i,1} = p(N_{t+1} = j | N_t = i)$$

For example $p_{2:6}^{0,1}$ is the probability of going from the state $N(010) = 2$, or $\{\bar{E}_1, E_2, \bar{E}_3\}$ at time 0, to the state where $N(110) = 6$, or $\{E_1, E_2, \bar{E}_3\}$ at time 1. Note the change in notation in going from C-I to the Markov formalism: $p_{i:j}$ means the probability of event i given event j, while $p_{i:j}^{t,k}$ means the probability of transition to state j from state i in a time interval

Downloaded by [75.94.224.40] at 17:26 07 July 2013

of k. The form of a general 3-event Markov matrix is:

$p_{i:j}^{t,t}$	x^{t+1}	(000)	(001)	(010)	(011)	(100)	(101)	(110)	(111)
	$N_{t+1} = j$	0	1	2	3	4	5	6	7
x^t	$N_t = i$								
(000)	0	$p_{0:0}$	$p_{0:1}$	$p_{0:2}$	$p_{0:3}$	$p_{0:4}$	$p_{0:5}$	$p_{0:6}$	$p_{0:7}$
(001)	1	$p_{1:0}$	$p_{1:1}$	$p_{1:2}$	$p_{1:3}$	$p_{1:4}$	$p_{1:5}$	$p_{1:6}$	$p_{1:7}$
(010)	2	$p_{2:0}$	$p_{2:1}$	$p_{2:2}$	$p_{2:3}$	$p_{2:4}$	$p_{2:5}$	$p_{2:6}$	$p_{2:7}$
(011)	3	$p_{3:0}$	$p_{3:1}$	$p_{3:2}$	$p_{3:3}$	$p_{3:4}$	$p_{3:5}$	$p_{3:6}$	$p_{3:7}$
(100)	4	$p_{4:0}$	$p_{4:1}$	$p_{4:2}$	$p_{4:3}$	$p_{4:4}$	$p_{4:5}$	$p_{4:6}$	$p_{4:7}$
(101)	5	$p_{5:0}$	$p_{5:1}$	$p_{5:2}$	$p_{5:3}$	$p_{5:4}$	$p_{5:5}$	$p_{5:6}$	$p_{5:7}$
(110)	6	$p_{6:0}$	$p_{6:1}$	$p_{6:2}$	$p_{6:3}$	$p_{6:4}$	$p_{6:5}$	$p_{6:6}$	$p_{6:7}$
(111)	7	$p_{7:0}$	$p_{7:1}$	$p_{7:2}$	$p_{7:3}$	$p_{7:4}$	$p_{7:5}$	$p_{7:6}$	$p_{7:7}$

Each row contains the probabilities of the system going from state i to state j at the next step. Each row must total to 100% because something must happen at every step even if it is only that the system does not change state. The probabilities that the system is unchanged after a transition lie on the diagonal where i = j. The larger these values, the slower the system is to change. These are sometimes called virtual, or internal, transitions, while the others are termed real transitions. The probabilities below the diagonal may be set to zero if events are irreversible (i.e., cannot "un-occur"). A Markov matrix for n events will have 2^{2n} state transition probabilities. If reversibility is prevented, $(2^n - 1)2^n/2$, probabilities below the diagonal may be set to zero. Even with this restriction, data requirements can be enormous for a Markov model. For example, the specification of a modest 6-event model would require $2^{12} = 4096$ entries for a full Markov matrix and $4096 - (2^6 - 1)2^6/2 = 2080$ entries if reversibility were prohibited. This greatly exceeds the requirements of the traditional C-I described earlier which needed at most $(2n^2 - n)$ probability estimates, only 60 estimated probabilities for a six-variable game (i.e., the off-diagonal elements of the occurrence and nonoccurrence matrices). Thus, while the Markov matrix allows a much richer model, the data requirements are great.

Downloaded by [75.94.224.40] at 17:26 07 July 2013

SPECIFYING MARKOV PROBABILITIES

How can one reduce the number of estimates required so as to make a Markovian C-I practical? Two strategies deserve consideration. One adapts the traditional C-I framework. Instead of asking for time-independent conditionals, here one asks questions such as, "What is the probability of E_j at time t + 1, given E_i at t?" This gives a probability $p(E_j^{t+1}|E_i^t)$. Alternatively one could ask for the probability of E_j within n time periods of E_i. This probability could be used to approximate the per-step probability. That is, let $p_i = p(E_j^{t+n}|E_i^t)$ and use

$$p(E_j^{t+n}|E_i^t) = 1 - (1 - p(E_j^{t+1}|E_i^t))^n$$

so that

$$p(E_j^{t+1}|E_i^t) = 1 - e^{n^{-1} ln(1-p_i)}$$

In this way, the expert is only required to give second-order conditional probabilities instead of conditionals that are functions of three or more events. Now, however, the conditionals are explicitly time-dependent. The idea here is to keep the simple structure of the traditional C-I to input data. Below are the occurrence and non-occurrence matrices for an example two-event, time-dependent model.

Occurrence Matrix Non-occurence Matrix

$$\begin{bmatrix} p(E_1^{t+1}|E_1^t) & p(E_2^{t+1}|E_1^t) \\ p(E_1^{t+1}|E_2^t) & p(E_2^{t+1}|E_2^t) \end{bmatrix} \begin{bmatrix} p(E_1^{t+1}|\bar{E}_1^t) & p(E_2^{t+1}|\bar{E}_1^t) \\ p(E_1^{t+1}|\bar{E}_2^t) & p(E_2^{t+1}|\bar{E}_2^t) \end{bmatrix}$$

$$\begin{bmatrix} 1. & .1 \\ .3 & 1. \end{bmatrix} \qquad \begin{bmatrix} .15 & .2 \\ .4 & .35 \end{bmatrix}$$

Note that the diagonals of the non-occurrence matrix may be non-zero, unlike the original game where it was logically impossible to observe an event both occurring and not occurring. These time-dependent diagonal probabilities are now the marginal probabilities of an event occurring during the next time period. If they are zero, the events can never occur. Likewise, non-unity in the occurrence matrix here means that an event may un-occur. This structure allows models wherein events can occur, un-occur, or reverse. This model requires only 2n more probabilities than the traditional C-I game, since the diagonals must now be estimated.

Downloaded by [75.94.224.40] at 17:26 07 July 2013

The probabilities of *event* transitions will be used to automatically construct the probabilities of *state* transitions that are used in Markov models. These state transition probabilities are of the form $p_{t:j}^{t,1} = p(N_{t+1} = j|N_t = i)$. The state transition probabilities will be approximated by the intersection of their constituent event probabilities. That is,

$$\begin{aligned} p_{i:j}^{t,1} &= p(N_{t+1} = j|N_t = i) \\ &= p((x_1^{t+1}, \ldots, x_n^{t+1})|(x_1^t, \ldots, x_n^t)) \\ &\approx \prod_{k=1}^{n} p(x_k^{t+1}, x_k^t) \end{aligned}$$

where

$$p(x_k^{t+1}, x_k^t) = \begin{cases} p(E_k^{t+1}|E_k^t) & \text{if} \quad x_k^{t+1} = 1 \quad \text{and} \quad x_k^t = 1 \\ p(E_k^{t+1}|\bar{E}_k^t) & \text{if} \quad x_k^{t+1} = 1 \quad \text{and} \quad x_k^t = 0 \\ p(E_k^{t+1}|\bar{E}_k^t) & \text{if} \quad x_k^{t+1} = 0 \quad \text{and} \quad x_k^t = 1 \\ p(E_k^{t+1}|E_k^t) & \text{if} \quad x_k^{t+1} = 0 \quad \text{and} \quad x_k^t = 0 \end{cases}$$

For example, in a 2-event model, the approximation is

$$p_{0:0}^{t,1} \equiv p(\bar{E}_1^{t+1}|\bar{E}_1^t)p(\bar{E}_2^{t+1}|\bar{E}_2^t) \tag{4}$$

These probabilities can be computed from the occurrence and non-concurrence probabilities using the two relations

$$p(\bar{E}_j^{t+1}|E_i^t) = 1 - p(E_j^{t+1}|E_i^t)$$

$$p(\bar{E}_j^{t+1}|\bar{E}_i^t) = 1 - p(E_j^{t+1}|\bar{E}_i^t)$$

Given this approximation, the Markov state transition matrix **P** for the example 2-event, the dependent model is:

$p^{t,1}_{i:j}$	$\mathbf{x}^{t+1}$	(00)	(01)	(10)	(11)
$\mathbf{x}^t$ $N_t = i$	$N_{t+1} = j$	0	1	2	3
(00)	0	$.85x.65$	$.85x.35$	$.15x.65$	$.15x.35$
(01)	1	$.85x.00$	$.85x1.0$	$.15x.00$	$.15x1.0$
(10)	2	$.00x.65$	$.00x.35$	$1.0x.65$	$1.0x.35$
(11)	3	$.00x.00$	$.00x1.0$	$1.0x.00$	$1.0xt1.0$

or

$$\mathbf{P} = \begin{bmatrix} .55 & .30 & .098 & .052 \\ .00 & .85 & .00 & .15 \\ .00 & .00 & .65 & .35 \\ .00 & .00 & .00 & 1.00 \end{bmatrix}$$

Note that the rows correctly sum to 100% so that this matrix is amenable to Markov analysis. One transition of particular interest occurs in the bottom row. The 100% probability means that once the system reaches that state, it will stay in it forever. Such a state is called a trapping state. A model may have none, one, or several trapping states.

In this example, eight event transition probabilities were used to generate sixteen state transition probabilities. The technique is even more beneficial when more events are to be modeled. One could still permit the expert to edit the resulting matrix to reflect specific insights that he or she may have (e.g., to adjust one or more of the transition probabilities.).

This method does not take into account the non-diagonal probabilities from the occurrence and non-occurrence matrices used in the traditional C-I analysis. One way to do this is to weight the diagonal transitions (used above) with these cross-impact terms. For example, in the 2-event example used above, equation (4) could be written as:

$$p^{t,1}_{0.0} \equiv \frac{1}{2}[p(\bar{E}_1^{t+1}|\bar{E}_1^t)p(\bar{E}_2^{t+1}|\bar{E}_2^t) + p(\bar{E}_1^{t+1}|\bar{E}_2^t)p(\bar{E}_2^{t+1}|\bar{E}_1^t)] \qquad (5)$$

where the second term represents the cross-impact terms. Weightings other than one half could be used. Equation (4) is easier to compute, but equation (5) considers the cross-impact.

The approach just described essentially ignores higher-order interactions among events. It may be worthwhile to adapt Monte Carlo simulation to generate state transition probabilities so that the Markov time-

Downloaded by [75.94.224.40] at 17:26 07 July 2013

Downloaded by [75.94.224.40] at 17:26 07 July 2013

dependent model incorporates higher-order event interactions. For instance, approximations similar to those used in the traditional C-I analysis could be added to the computation of the transition probabilities.

The second strategy to reduce the number of estimates required is to estimate the state transition probabilities directly. That is, one asks the expert to assess the likelihood of transitioning from some given state at time to each feasible successor state by the t+1. Consider the AIMTECH example introduced earlier. Suppose ANA has occurred but COM and ADV have not. The expert is then asked to estimate the likelihood that at time t+1 the system will transition to one of the feasible states: ANA only (no change); ANA and COM; or to ANA, COM and ADV. This works like Martino's (1983) temporal strategy for a more traditional C-I by considering only the more plausible states. It can further simplify estimation by assuming constant state transition probabilities for some or all time periods. Eymard (1977) used a combined strategy in which event occurrence probabilities were estimated for selected prior event combinations. Kaya et al. (1979) reduced the event set by retaining only those events that had the greatest total impact to and from other events. The next section addresses state transitions.

State Transition Probabilities

The probability that the system will make a transition from any state i at time t to any state j after n steps is given by the transition matrix $\mathbf{P}^{(t,n)}$,

$$\mathbf{P}^{(t,n)} = \mathbf{P}^{(t,1)}\mathbf{P}^{(t+1,1)} \ldots \mathbf{P}^{(t+n-1,1)} \tag{6}$$

In the special case that $\mathbf{P} = \mathbf{P}^{(t,1)}$ for all values of t, $\mathbf{P}^{(t,n)} = \mathbf{P}^n$, and the n-step transition matrix is obtained by multiplying the matrix $\mathbf{P}$ by itself n times. Using the numbers from the last illustration, the n-step transition matrix can be shown to be

$$\mathbf{P}^n = \begin{bmatrix} .55^n & .85^n - .55^n & .98(.65^n - .55^n) & 1 - .85^n - .98(.65^n - .55^n) \\ 0 & .85^n & 0 & 1 - .85^n \\ 0 & 0 & .65^n & 1 - .65^n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For this example, one transition of interest in that from state 0 (no events occurred) to state 3 (both events occurred, trapping). The proba-

Table 2: Illustration of $p_{0:3}(n)$ and $f_{0:3}(n)$ for selected n

n	$p_{0:3}(n)$	$f_{0:3}(n)$
0	0	
1	0.052	0.052
2	0.160	0.108
3	0.280	0.120
4	0.393	0.113
5	0.492	0.099
6	0.576	0.084
7	0.646	0.070
8	0.704	0.058
9	0.753	0.048
...	...	...
19	0.954	0.008
20	0.961	0.007

Downloaded by [75.94.224.40] at 17:26 07 July 2013

bility of this transition in n-steps is represented by element $p_{0:3}(n)$ in the above matrix $\mathbf{P}^n$. In other words, $p_{0:3}(n)$ is the probability that we are in state 3, n steps after being in state 0. Note that we either may have arrived at this state earlier and remained, or just arrived at state 3 on the *nth* step. We call the latter probability the *first passage time*, the time to *first* enter a particular state and denote it $f_{i:j}^{(n)}$. The formula for the first passage probability for a system with *constant* transition probabilities, $p_{i:j}^{t,1}$, is

$$f_{i:j}^{(n)} = p_{i:j}(n) - \sum_{k=1}^{n-1} f_{i:j}^{(k)} p_{j:j}(n-k) \tag{7}$$

Table 2 lists the values of these probabilities for several values of n. Note that as n becomes large, $p_{0:3}(n)$ approaches 1 and $f_{0:3}^{(n)}$ approaches 0. The values of $p_{0:3}(n)$ can be used to determine confidence intervals for trapping. For instance, after 5 steps it is almost 50% certain that the system will have reached trapping. To be 95% certain of this transition, the system would have to make at least 19 steps. From the table it can be seen that, for instance, the system is most likely to make the 0-3 transition on the 3rd step, where the probability is 12%.

The mean number of steps or expected time for this 0-3 transition can

Downloaded by [75.94.224.40] at 17:26 07 July 2013

be calculated from a weighted sum of the first passage probabilities. Let $T_{i:j}$ be the random first passage time from state i to state j. Its mean is

$$\mu(T_{i:j}) = \sum_{n=1}^{\infty} n f_{i:j}^{(n)}$$

For example, summing the first 50 terms of this equation for the $\mu(T_{0:3})$ yields 7.28 – the average number of periods to get from state 0 to state 3 for the first time.

Final State

The probabilities for final states can be found by raising $\mathbf{P}$ to some large n. In this example,

$$\lim_{n\to\infty}\mathbf{P}^n = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since all the rows are the same, the final probabilities are independent of starting state; after many transitions the system will be in state 3, trapping, regardless of initial state. If the reversal (unoccurrence) of states were allowed, it would have been possible to have a system without trapping states. The rows of $\mathbf{P}$ would still sum to 1 but every probability would be less than 1.

State Holding Time

Another statistic of interest is the holding time H_i for a recurring state i. That is, after entering a state i that has a non-zero virtual transition probability, how long can one expect the system to stay in that state? The probability that the system will stay in a recurring state n steps after entering is given by

$$p(H_i = n) = (1 - p_{i:i}^t)(p_{i:i}^t)^{n-1}$$

The number of transitions n that a state will "hold" the process in geometrically distributed with a parameter that depends only on the $p_{i:i}^t$ probability. The mean and variance of this holding time are given, respectively, by

$$\mu(H_i) = 1/(1 - p^t_{i:i})$$

$$\sigma^2(H_i) = p^t_{i:i}/(1 - p^t_{i:i})^2$$

For the 2-event example $\mu(H_i), i = 0, 1, 2$ work out to be 2.22, 6.67, and 2.86, respectively, while the variances of the holding times are 2.72, 37.78, and 5.31, respectively.

Downloaded by [75.94.224.40] at 17:26 07 July 2013

Scenario Probabilities

The probabilities of every scenario can be calculated to find the most likely one. Since virtual transitions merely delay the time for the scenarios, these are eliminated for this calculation. The matrix $\mathbf{P}_R$ of only real transitions is shown below:

$$\mathbf{P}_R = \begin{bmatrix} .00 & .30 & .098 & .052 \\ .00 & .00 & .00 & .15 \\ .00 & .00 & .00 & .35 \\ .00 & .00 & .00 & .00 \end{bmatrix}$$

When normalized row-by-row so that rows total 100%, this becomes:

$$\begin{bmatrix} .00 & .66 & .22 & .12 \\ .00 & .00 & .00 & 1.0 \\ .0 & .00 & .00 & 1.0 \\ .00 & .00 & .00 & .00 \end{bmatrix}$$

There are only three possible scenarios for going from state 0 to state 3 – 0 to 3, 0 to 1 to 3, and 0 to 2 to 3. The probabilities of each trajectory are simply the products of the constituent state transition probabilities:

Trajectory	Probability
0 to 3	.12 = 12%
0 to 1 to 3	.66x1.0 = 66%
0 to 2 to 3	.22x1.0 = 22%

Thus the most likely scenario is the one that starts in state 0, followed by states 1 and then 3. This path can be expected to take 2.22 + 6.67

+ 2 = 10.89 steps – that is, it remains in state 0 for an average of 2.22 steps before moving on to state 1, where it remains for an average of 6.67 steps, plus the two transitions themselves (0 to 1, 1 to 3).

The succession among Markov states may be thought of as a "path scenario" which provides future history in which certain events occur, followed by other events, as time unfurls. In dealing with any substantial number of events over several time periods, the probabilities for any scenario become vanishingly small. Some C-I programs provide results consisting of sets of Monte Carlo simulated event occurrences over time (e.g., the XIMPACT program, Enzer and Leschinsky, 1986). The user examines these to infer "likely" scenarios. Mitchel et al. (1977) review scenario generation procedures, comparing the use of linear programming, mixed-integer linear programming, and simulation to identify the most likely event combinations. Martino and Chen (1978) try cluster analysis to combine relatively similar scenarios to get a better overall representation of likely future patterns.

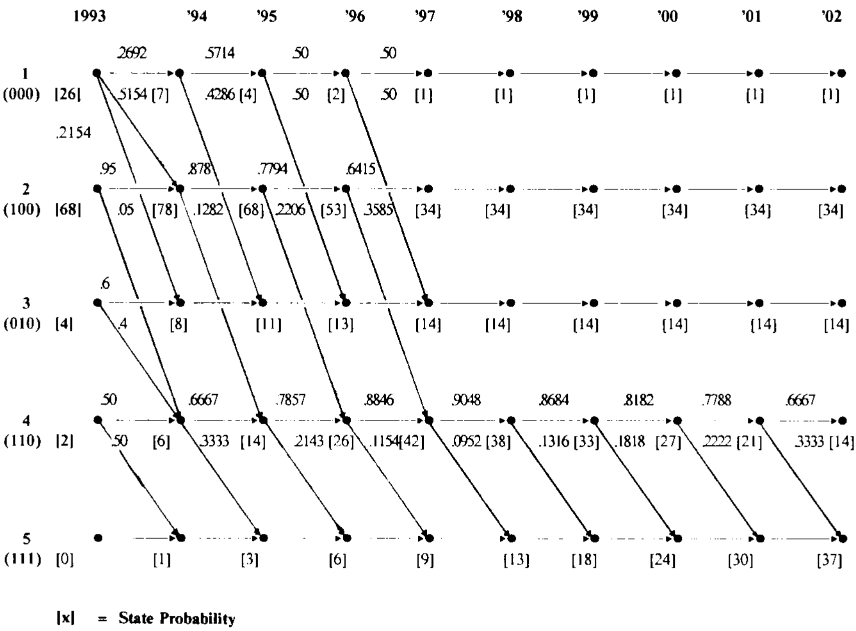
Downloaded by [75.94.224.40] at 17:26 07 July 2013

Markov analysis can answer a variety of questions about a system. It can tell us the expected number of transitions required to get to any state from an initial state (or a distribution of initial states). Probabilities for the system to be in a particular state after a given number of transitions from another state can be calculated and the expected number of periods to make the transitions, as well the variances of the number of periods can be found (Howard, 1971: pp. 201-205). These can be used to predict behavior through the use of confidence intervals, both in terms of states and of whole trajectories (i.e. most likely scenarios) over time. The total time spent in each state can be calculated, and if there are virtual transitions, the holding time for those states can be calculated. For most of the statistics, the calculations are matrix operations that can be performed readily by computers.

AN ILLUSTRATIVE MARKOV C-I: AIM-TECH

Figure 1 provides Markov transitions that reproduce the data from Table 1. The probabilities are provided above each state time line, and numbers in [ ] represent the state probability at each time. Event 3, ADV, can only occur after both Events 1 (ANA) and 2 (COM) have occurred; no events can un-occur. Because of the ADV constraint, only 5 of the $2^3 = 8$ theoretical states are modelled (the other 3 states cannot occur). These feasible states are numbered as described in the footnote to Figure 1.

Figure 1: Markov representation of data in Table 1 (Missing entries are zero)



key: $E_1 = ANA, E_2 = COM, E_3 = ADV$			
* State	1	(000)	No Events have occurred
	2	(100)	Only ANA has occurred
	3	(010)	Only COM has occurred
	4	(110)	Both ANA and COM has occurred
	5	(111)	All events have occurred

Downloaded by [75.94.224.40] at 17:26 07 July 2013

Downloaded by [75.94.224.40] at 17:26 07 July 2013

The transition probabilities shown in Figure 1 vary with time. The figure shows both the transitions and the dynamic probabilities of being in a particular State, given a starting probability distribution over the states of $\mathbf{P}^{1993} = (.26, .68, .04, .02, 0)$. The figure summarizes a wealth of information. For instance, note that Event 1 occurs by 1994, or never since there are no transitions from State 1 to State 2 or from State 3 to State 4 after 1994. The probabilities of States 2 and 4 limit the eventual probability that State 5 can attain to a maximum probability of .37 as noted in the figure. The probability of State 2 diminishes between 1994 and 1997 as the probability of transition to State 4 (ANA and COM achieved) increases. This latter State then limits the possibilities for State 5 thereafter. Similarly Event 2 must occur by 1997 (i.e, $\mathbf{P}_{1:3}$ and $\mathbf{P}_{2:4}$ are zero after 1997), so the population of State 4 declines after that year. Also, States 1, 2, and 3, do not change after 1997. The whole profile is stagnant from 2002. To the extent that one believes the model, it would argue against concentrating R&D resources on achieving Event 3 unless Event 2 can again become possible.

The state probabilities can be computed in the following way, using the data from Figure 1. For the transition from 1993 to 1994 we observe that

$$\mathbf{P}^{1993,1} = \begin{bmatrix} .2692 & .5154 & .2154 & 0 & 0 \\ 0 & .95 & 0 & .05 & 0 \\ 0 & 0 & .6 & .4 & 0 \\ 0 & 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The 1994 probabilities are then computed using the formula, $\mathbf{p}^{1994} = \mathbf{p}^{1993}\mathbf{P}^{1993,1}$ (the two probability vectors $\mathbf{p}^{1993}$ and $\mathbf{p}^{1994}$ are *row* rectors). Each period can be computed from the last in this way. Several period transitions can be computed using Equation (6): for instance, $\mathbf{P}^{1993,3} = \mathbf{P}^{1993,1}\mathbf{P}^{1994,1}\mathbf{P}^{1995,1}$.

The impacts of resource allocation on the probabilities of occurrence of the 3 events can be further examined. As resources decrease, the diagonal probabilities, $\mathrm{p}_{i:i}^{t,1}$, increase at the relative expense of the super-diagonal terms, $\mathrm{p}_{i:j}^{t,1}$ for $j \succ i$; the reverse occurs as resources increase. Consider the evolution of $\mathbf{p}^{1996}$ if the resources were reduced. Then the prospects for COM and ADV in 1996 would be reduced accordingly. Most

Downloaded by [75.94.224.40] at 17:26 07 July 2013

importantly, this impact would reduce the State 4 probability – the base for 1997 and later year chances of attaining State 5.

This does not exhaust the analysis possibilities using the Markov formulation. We can examine the last column of the transition matrix $\mathbf{P}^{1993,9}$, to obtain the probabilities of attaining ADV by 2002 from the intitial states (Matrix not given here). These probabilities, (.2041, .4184, .3377, .9221,1.) indicate that if we are in State 1 in 1993, we have a 20.41% chance of reaching ADV by 2002; our greatest chance occurs if we are in State 4, with a 92.21% chance (of course, if we start with ADV we are certain to be there in 2002). Averaged over the distribution of starting states we get 37% chance of success as noted on Table 1.

Provided that we make it to ADV, how long will it take? This is a question of the first passage time, from starting State i to State 5. This can be computed using a generalization of Equation (7) for non-constant transition probabilities.

$$f_{i:j}^{(n)} = p_{i:j}^{1993,n} - \sum_{k=1}^{n-1} f_{i:j}^{(k)} p_{j:j}^{1993+k,n-k} \tag{8}$$

The calculation is simplified since $p_{5:5} \equiv 1$. The first passage distribution is obtained through application of Equation (8) and normalization by the conditional probability of actually attaining ADV. These distributions are summarized in Table 3. The mean first passage times can be computed from these values: (6.7,6.4, 4.2, 2.5) (years). Weighted over the starting State distribution yields an expected first passage time of 6.3 years, since the probabilities are concentrated initially on States 1 and 2. Clearly there are many possibilities for analysis when the Markov model is used.

Contrast the Markov analysis with the traditional C-I. The latter provided revised cumulative probabilities for the conclusion of the period – 2002 (i.e., marginal probabilities for ANA, COM, and ADV). Traditional C-I can approximate time steps (Enzer and Leschinsky, 1986), but causality implied is not explicitly modeled. Consideration of the C-I matrices would also highlight the extreme dependence of Event 3 (ADV) on the other 2 events. In contrast, the Markov approach provides information on the process throughout the time period. This could be helpful in timing the allocation of scarce resources for the years wherein they might have greatest impact. For instance, one might probe whether increased work

Table 3: Conditional First Passage Distributions for AIM-TECH Example

First Passage to State 5 in Year	Starting State 1	2	3	4
1994	0.	0.	0.	.5442
1995	0.	.0394	.3909	.1789
1996	.0694	.0795	.1700	.0778
1997	.0854	.840	.0720	.0329
1998	.1196	.1128	.0520	.0138
1999	.1512	.1426	.0657	.0300
2000	.1814	.1710	.0788	.0307
2001	.1814	.1710	.0788	.0307
2002	.2116	.1996	.0992	.0421

Downloaded by [75.94.224.40] at 17:26 07 July 2013

on COM in the early years (Table 1) could speed up its development relative to ANA. Likewise, one might examine further the need for sizable investment in ADV prior to the early 1990s.

CONCLUSIONS

Cross-impact analysis can be expressed by using Markov processes: with extensions, both trends, and events can be incorporated in the analysis. Trends can only be represented easily when they are taken over a finite range. The formalism is fairly straightforward. It requires the estimation of state transition probabilities from one time interval to the next. The selection of the actual time interval is arbitrary, depending on the needs of the analysis. While this approach offers a number of improvements over the usual C-I analysis, it can require a large amount of information to construct the transition matrices, especially where trends are involved. When the full transition matrix is specified (most practical when the transition probabilities are constant), Monte Carlo simulation is not needed. Markov C-I offers great potential for richer modeling of technological change and impact processes.

References

[] Enzer, S. and D. Leschinsky 1986. *XIMPACT User's Guide and Manual*, Release 1.0. Los Angeles: University of Southern California.

[] Halverson, T., J.J. Swain, and A.L. Porter 1989. "Analysis of the Cross-Impact Model," Technology Policy and Assessment Center Technical Report. Atlanta, GA: Georgia Institute of Technology.

[] Howard, R.A. 1971. *Dynamic Probabilistic Systems, Vol. 1: Markov Models*. John Wiley & Sons.

[] Martino, J.P. 1983. *Technological Forecasting for Decision Making* (2nd Ed). New York: North Holland.

[] Porter, A.L., F.A. Rossini, S.R. Carpenter and A.T. Roper 1980. *A Guidebook for Technology Assessment and Impact Analysis*. New York: North Holland.

[] Reitman, W., R.M. Weischedel, K.R. Boff, M.E. Jones, and J.P. Martino 1985. "Automated Information Management Technology (AIM-TECH): Considerations for a Technology Investment Strategy." Air Force Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio (AFAMRL-TR-85-042). Xu, H. *Cross-impact Models for Technology Forecasting and Impact Assessment* 1990. M.S. Thesis, Technology and Science Policy Program. Georgia Institute of Technology, Atlanta, GA.□

Downloaded by [75.94.224.40] at 17:26 07 July 2013